WRITTEN HOMEWORK #3, DUE JAN 27, 2010

(1) Let D be the circle with center at $(0, 2)$ and radius 2. Evaluate the double integral

$$\iint_D y\, dA.$$

Hint: Do not try to write polar inequalities which define D. Instead, begin by making a change of variables, from y to $y-2$, which translates the center of the circle to the origin. If we call this new disc D', then we have

$$\iint_{D'} (y+2)\, dA = \iint_D y\, dA.$$

(Convince yourself that this equation is true by thinking about the volumes of the solids which lie under the graphs of $z = y+2, z = y$ over D', D respectively.)

(2) (Chapter 16.4, #14) Let D be the region in the first quadrant between the graphs of $x^2 + y^2 = 4, x^2 + y^2 = 2x$. Evaluate

$$\iint_D x\, dA.$$

(Hint: Do not try to write polar inequalities for D. Instead, express the double integral you want to evaluate as a difference of double integrals over two different regions. You might want to think about the proceeding problem to help you evaluate one of those integrals.)

(3) (Chapter 16.4, #25) Find the volume above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

(4) (Chapter 16.5, #12) A lamina occupies the part of the disc $x^2 + y^2 \leq 1$ which lies in the first quadrant. Find the center of mass of the lamina if the density of the lamina at a given point is proportional to the square of the distance of that point from the origin.

(5) (Chapter 16.5, #18) For the same lamina as 16.5.12, find the moments of inertia I_x, I_y, I_0. (The moment I_0 is the moment of inertia about the origin, and is equal to $I_x + I_y$.) You should assume that the density function is given by $\rho(x, y) = x^2 + y^2$.